

Stochastic Hydrology 622 U3600

Homework 5

Due 16/05/2007

1. Let ω be selected at random from the interval $S = [0, 1]$, where we assume the probability that ω is in a subinterval of S is equal to the length of the subinterval. Determine in what senses the following random sequences converge:
 - (1) $X_n(\omega) = e^{-n\omega}, n \geq 0$
 - (2) $X_n(\omega) = \sin(\omega + \frac{1}{n}), n \geq 1$
 - (3) $X_n(\omega) = \cos^n(\omega), n \geq 0$
 - (4) If the preceding sequences converge, what are the limits?
2. Let W_0, W_1, \dots be independent, normal random variables with mean 0 and variance 1. Let $X_{-1} = 0$ and $X_n = 0.9X_{n-1} + W_n, n \geq 0$.
 - (1) Generate and plot 10 realizations of the random process $\{X_1, X_2, \dots, X_{1000}\}$.
Can you observe any property of stochastic convergence from these data?
 - (2) In what sense does X_n converge as n approaches infinity?
3. Construct a simple scaling, dimensionless design hyetograph with 24 time increments using the storm rainfall data in the file SSGM_rainfall_data.xls. [Note: You may contact Ru-Jen for a computer program.]